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EXTREMA PROBLEMS SOLVED IN GEOGEBRA

39.1 INTRODUCTION

The future bachelors and engineers have to understand not only theoretical background of math but also its applicability to real problems. The courses of mathematics at our university have been enlarged by applied examples. We used the main advantages of GeoGebra, its playfulness and interactivity, to get involved the students for studying mathematics. They learn to formulate the problem, create mathematical description of the problem, model different situations with respect to the initial data.

The paper was prepared with the help of the following references, [1–5].

39.2 EXTREMA PROBLEM

With the extremal problems we meet since the times of antiquity and ancient Greece. It was primarily geometric problems. A typical example is the so-called isoperimetric problem: find a closed curve of the known length, which encloses a surface with maximum area. The solution is, of course, a circle. Later, the extremal problems began to appear in other mathematical disciplines, in particular in connection with the development of mathematical analysis and especially with differential calculus.

Extremal problems can be found not only in mathematics. In fact, almost all human activities are subject to a certain form of intuitive search solution of some extremal problems. We are trying to get somewhere in the shortest possible time, travel the shortest distance, buy as many stuff for the least money, etc.

In mathematics the extremal problems represent a nice example of real usability of differential calculus. GeoGebra allows us to obtain a geometric view of the problem, it allows us to dynamically model and estimate the solution. The estimated solution can then be compared with the solution obtained by mathematical formulation of the extremal problems, i.e. we are almost looking for an extremum of some function.

39.3 EXAMPLE: MINIMAL TRANSPORTATION COSTS

Example: A factory lies near a road passing through a town. The minimum distance between the factory and the road is a *km*, and the direct distance between the factory and town is b *km*.
A new road has to be built to connect the factory and the old road (at point X) as shown on the following map. Transportation costs are 25 € on the new road and 15 € on the old road per *km*. Where should anybody places the connecting point X to minimize the transportation costs?

39.3.1 The task scheme

On the following picture one can find the geometric formulation of the problem.

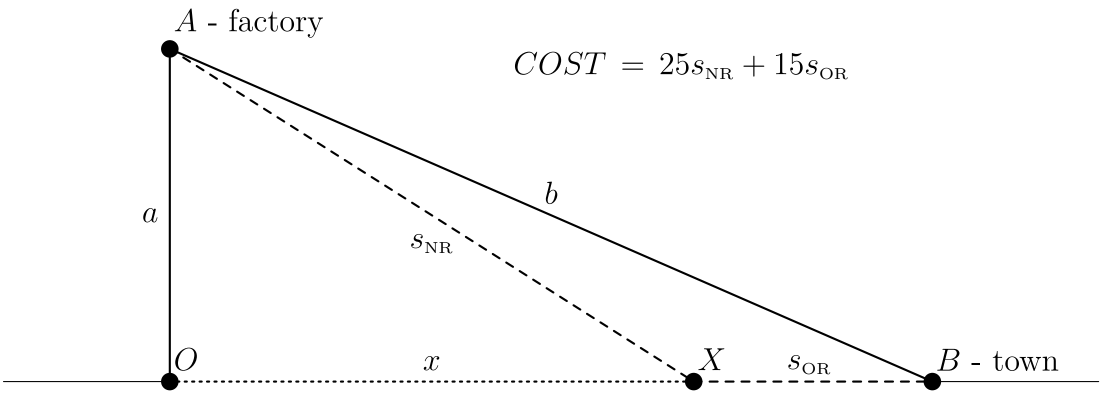

Fig. 39.1 Extrema problem – transportation costs

Source: own elaboration

Let's denote the length of the new road s_{NR} and the old road s_{OR}. It holds:

$$s_{\mathrm{NR}} = \sqrt{a^2 + x^2}, \quad s_{\mathrm{OR}} = \sqrt{b^2 - a^2} - x. \tag{39.1}$$

The transportation costs are:

$$f = 25s_{\mathrm{NR}} + 15s_{\mathrm{OR}} = 25\sqrt{a^2 + x^2} + 15\left(\sqrt{b^2 - a^2} - x\right). \tag{39.2}$$

In the next subsection we show how to construct the geometric dynamic model of our problem and allow the reader to estimate the solution by this tool. In the subsection 39.3.3 we study the function (39.2) and we find exact solution by means of internal GeoGebra command `Extrem`.

An analytic solution of the problem is presented in subsection 39.3.4 in the form typical for common mathematical lecture held without computer.

39.3.2 Interactive tool in GeoGebra

At first, the task is reformulated from the geometric point of view, the parameters a and b are represented by the sliders (see Fig. 39.2).

1.		Insert *Slider* `a` from 0 to 10 with increment 0.1, change color to green. Set value to `4`.
2.		Insert *Slider* `b` from 0 to 10, with increment 0.1, change color to blue. Set value to `10`.
3.		Create Point `A` by entering `A=(0,a)` into the *Input Bar*.
4.		Create the *Circle* `c` with center `A` and radius `b`.
5.		*Intersect* between circle `c` and `XAxis`. There are two intersect points. The right one One of them is `B`.
6.		Create Point `O` by entering `O=(0,0)` into the *Input Bar*.
7.		Create *Line* by points `O` and `B` and rename it `road`.
8.		Create *Segment* `AO` between points `A` and `O`, show value of this segment and change color to green. Is this value the same as value of Slider `a`?
9.		Create *Segment* `AB` between points `A` and `B`, show value of this segment and, change color to blue. Is this value the same as value of Slider `b`?
10.		Create *Segment* `OB` between points `O` and `B`, show value of this segment.
11.		Change the value of the sliders and check the values of segments and sliders with same color.

The position of point X given by a distance between X and the origin of the coordinate system is represented by the slider x_G. (see Fig. 39.2).

12.		Create *Slider* `x_G` from 0 to `sqrt(b^2-a^2)`, change color to red. Set value to `5`.
13.		Create Point `X` by entering `X=(x_G,0)` into *Input Bar*, change color to red.
14.		Create *Segment* `AX` between points `A` and `X`, show value of this segment.
15.		Create *Segment* `XB` between points `X` and `B`, show value of this segment.
16.		Create *Segment* `OX` between points `O` and `X`, show value of this segment and change color to red.
17.		Input `f_G=25*AX+15*XB` into the *Input Bar*. Drag and drop `f_G` from Algebra View to Graphics View.
18.		Change the value of the slider `s` and try to find minimal value of `f_G`.

In order to minimize the transportation costs we have to minimize the value f_G, which depends on the value of the slider x_G.

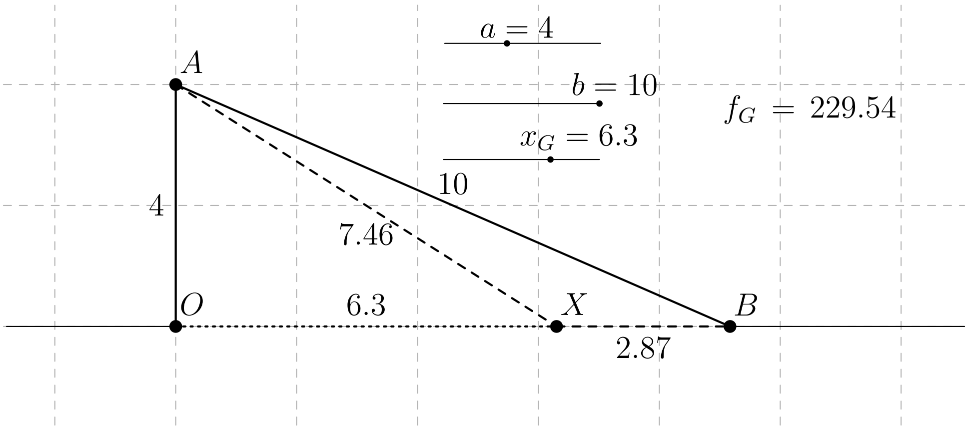

Fig. 39.2 Extrema problem – geometric point of view

Source: own elaboration

39.3.3 The "function" - mathematical model

Next step is to construct the symbolic representation of the task. Instead the concrete value of the slider x_G (respectively f_G) we introduce a variable x (respectively function $f(x)$), see Fig. 39.3.

19.	*View*	Open *Graphics View 2* for the and following steps.
20.		Input `f(x)=Function[25*sqrt(a^2+x^2)+15*(sqrt(b^2-a^2)-x),0,d]` into the *Input Bar*.
21.		Input `MinValue=Extrem[f,0,d]` into the *Input Bar*, change color to orange.
22.		Input `F=(x_G,f(x_G))` into the *Input Bar*, change color to red.

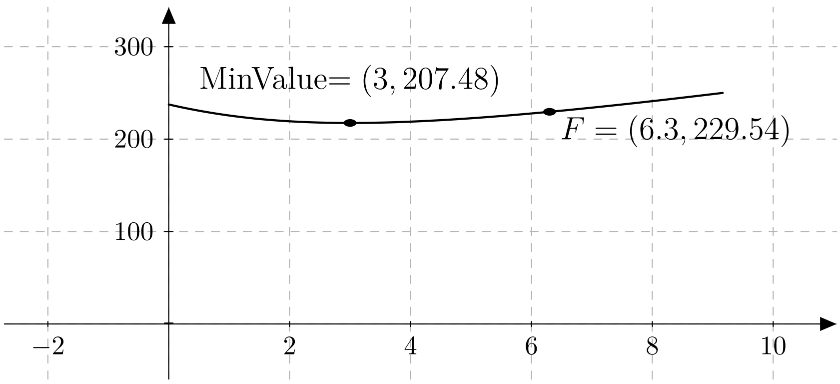

Fig. 39.3 Extrema problem – minimized function

Source: own elaboration

39.3.4 Analytic solution of the problem

Let's differentiate the function f:

$$f(x) = 25\sqrt{a^2 + x^2} + 15\left(\sqrt{b^2 - a^2} - x\right),$$
$$f'(x) = 25\frac{x}{\sqrt{a^2 + x^2}} - 15.$$

From the condition for the stationary point of the function f immediately follows:

$$f'(x) = 0 \quad \Rightarrow \quad 25\frac{x}{\sqrt{a^2 + x^2}} - 15 = 0 \quad \Rightarrow \quad x = \frac{3a}{4}.$$

CONCLUSION

We have a very positive feedback from the students with including of the extremals problems in mathematical lectures. GeoGebra is the best software for visualization of such problems and helps to better understanding of the calculus.

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5. `http://www.geogebra.org/en/wiki`

EXTREMA PROBLEMS SOLVED IN GEOGEBRA

Abstract: *The solving of applied problems is a reasonable way to gain better understanding of mathematical concepts, especially for future engineers. GeoGebra is a very useful tool for modeling these problems thanks to its interactivity.*

Keywords: *GeoGebra, extrema problems.*

EXTREMÁLNÍ ÚLOHY ŘEŠENÉ V PROGRAMU GEOGEBRA

Abstrakt: *Řešení aplikovaných úloh je jeden z vhodných způsobů pro lepší pochopení matematických konceptů především pro budoucí inženýry. Díky své interaktivitě je GeoGebra velmi užitečný nástroj pro modelování takových úloh.*

Klíčová slova: *GeoGebra, extremální úlohy.*

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