14

SAVING ENERGY IN LIGHTS

14.1 INTRODUCTION

In our system we consider as variables: date, geographic situation and height observer (zero for example).

Using these parameters we could solve the sun rise/set time, twilights (civil, nautical and astronomical), the hour of sun at mid-day local meridian [PMSL] and sun's height at any time.

The electric saving is our final target and we want to know at what time the streetlights must be switched on/off, but using mathematic formulas.

In our case, the engineering give a lot of automatic solutions (Fig. 14.1), such as to use photocells and servos in programmable machines to follow the sun path in the sky.

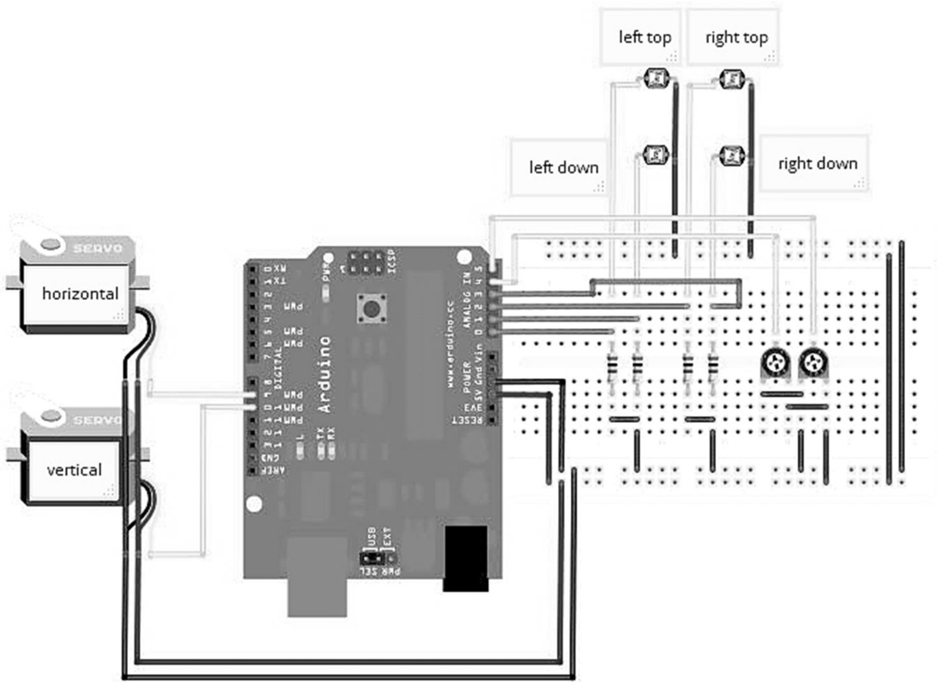

Fig. 14.1 Arduino solar tracker

Source: http://www.instructables.com/id/Arduino-Solar-Tracker/step4/The-circuitry/

Using these systems you can rich more than 7% of improvement over common following methods. However, all of these proceedings not let to do previous works management.

14.2 KNOWLEDGE OF NAUTICAL CONCEPTS AND STARS [3]

There are no doubt, that nautical knowledge using stars will be very useful to rich our final target, but not everybody have it.

The minimal basics concepts must be introduced here to understand how to solve the exercise. The sun it is one more star only, but near from the earth and very studied.

The first item could be the knowledge of celestial sphere or geometrical place on which a terrestrial observer projects the stars views.

The celestial sphere (Fig. 14.2) has many coincidences with the earth such us celestial north and South Pole, equator, meridians and parallels.

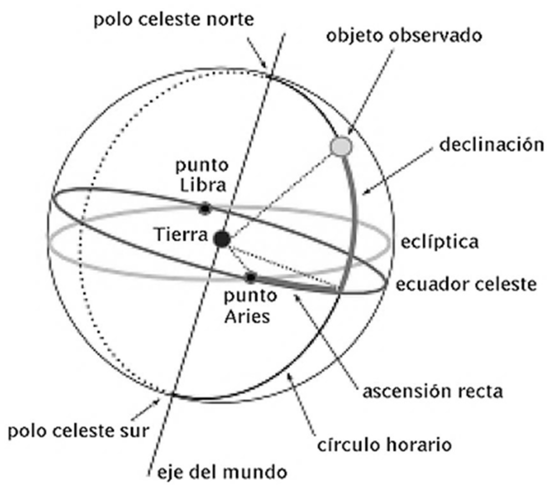

Fig. 14.2 Celestial Sphere

In the fact, the celestial sphere is a concentrical from the earth but bigger radius and revolves around the centre in function of date and hour.

It is possible to use a celestial coordinates based on latitude and longitude or declination and right ascension and establish transformations between both of them.

The fact, all of stars and sun exercises must be solved calculating the position triangle of the star. This position triangle is a spherical triangle and we can use Bessel's formulas to solve it. The triangle is created with 3 sides measured in degrees and 3 angles measured in degrees too.

Angles (Fig. 14.3):

P = Polar angle or angle in Pole. Is the eastern (E) angle or western (W) less than 180^0 formed between the local meridian (where the observer is situated) and the star's meridian. The same condition and terms but considering clockwise is called *"star's local hour angle (LHA)"* for an observer using English notation. In nautical and Spanish notation is [H*L] or [H☼L] were "H" = Hour; "*" = star and "L" = local. For the sun we can follow the same method where "H" = Hour; "☼" = the sun and "L" = local.

The notation followed for polar angle in our exercise will be west (+) azimuth clockwise and east (-).

When P is west and less than 180^0, **P** = [H☼L] = [H*L]

[*] Remember that our star (*) is the sun (☼) and * = ☼.

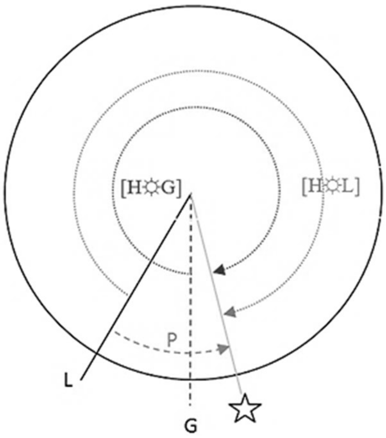

Fig. 14.3 Polar angle

The ***Polar Angle*** is the responsible of ***"The time"***. For example, at what time is the rising sun? To translate the angle to hours we shall divide between 15 degrees in 1 hour or average speed for the sun. If you want more accuracy, could be the day' sun speed calculated as difference between the hour of the sun in two consecutive hours but in final results it is not far (seconds) from first.

Example:

observer longitude is L= W 009^0 20' 36" and [H☼L] = 318^0 23' 46". Then P = 360^0 - [H☼L]

P = 41^0 33' 14" (East).

[H☼G] = [H☼L] + L➔ [H☼G] = 318^0 23' 46" + 009^0 20' 36" = 327^0 44'22"

Z = Azimuth (Fig. 14.4).

It is the angle between observer latitude meridian and vertical projection of the star over the horizon line.

A = Paralactic angle. Not used neither with nautical proposals nor even to calculate heights and star's positions.

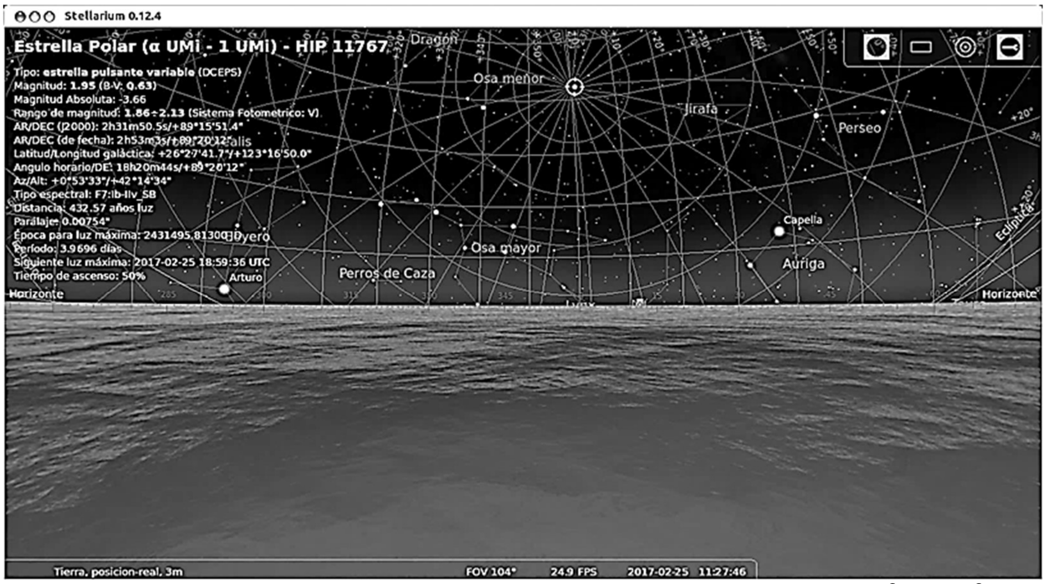

Fig. 14.4 Azimuth. Example: The north. Polaris star azimuth (Az) = 0^0 = 360^0

The sides of triangle (Fig. 14.5 and Fig. 14.6)*:*

$\emptyset_l$ = **colatitude** = 90^0 – latitude

$\emptyset_\delta$ = **codeclination or polar distance** = 90^0 – declination

(*) Signs criteria:

- The north hemisphere must be considered (+)
- The south hemisphere must be considered (–)
- Declination' sign must be consider in function of star's position respect the celestial equator circle. The declination of vernal point equinox Aries (spring equinox) point and Libra (autumn equinox) is zero. Upper positive and negative below. For instance, in the north hemisphere from March, 20^{th} to September, 23^{th} the declination is positive and negative in the south hemisphere.

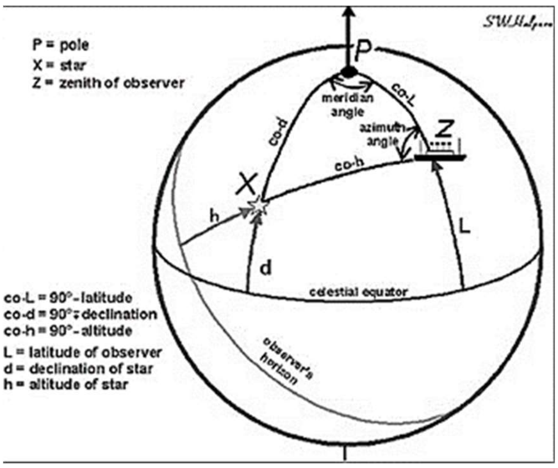

Fig. 14.5 Triangle of position

For date 22/02/2017, and north hemisphere (+), the sun's declination is (-) and codeclination will be 90^0– (–δ) = 90^0+δ. The discussion signs is (+) North hemisphere × (–) declination = (–). In the south will be (–) × (–) = (+); codeclination will be $90^0 – (+\delta) = 90^0 - \delta$.

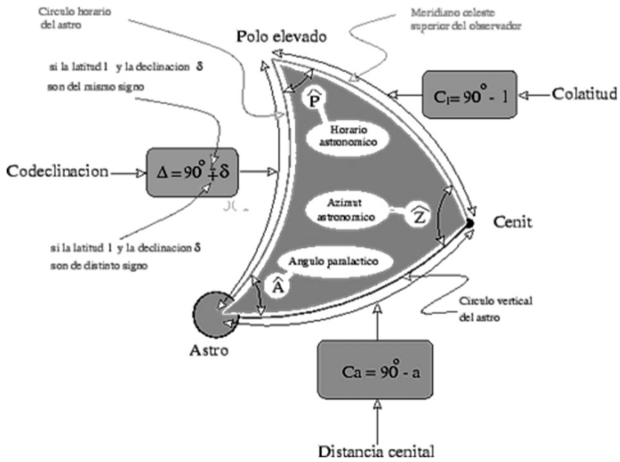

Fig. 14.6 Position's Triangle

$\varnothing_a$ = **coheight or zenithal distance** = 90^0 – sun's height (h). It is just like the same process than codeclination. If the sun is visible (upper earth horizon line) the sun's height is (+) and $\varnothing_h = 90^0 - (+h) = 90^0 - h$. In sunset or sunrise h = 0, and $\varnothing_h = 90^0$.
In Spanish notation height = altura, abbreviated "a" and $\varnothing_a = 90^0 - a$.
In twilights the sun's height is: civil (-6^0), nautical (-12^0) and astronomic (-18^0) and $\varnothing_{h,\,civil} = 96^0$, $\varnothing_{h,\,nautical} = 102^0$ y $\varnothing_{h,\,astronomic} = 108^0$
It should be possible to solve the triangle using a nautical almanac, because the values of different variables have been tabulated inside. But we need to explain any more if we want to solve it using mathematics' formulas.

Time's origin. Celestial calendar. Julian day (JD) [1]

It is evident, that the calendar used by the stars is different from calendar Gregorian, Muslim, Jew, Mayan, Egyptian, etc. The issue is that for the star must be a source of time, and this has caused many problems over the life of the man. A day does not exactly last a day and similarly passes over the years and the centuries.
As science has progressed, the need for "accuracy" in the calculations of the Sun and other stars have been profiling this source located in year 4713 B.C., i.e. in the -4713. From this source are counted as Julian day (JD).
Julian day [1] is the continuous count of days since the beginning of the Julian Period and is used primarily by astronomers.
The **Julian Day Number** [1] (**JDN**) is the integer assigned to a whole solar day in the Julian day count starting from noon Universal time, with Julian day number 0 assigned to the day starting at noon on January 1, 4713 BC, proleptic Julian calendar (November 24, 4714 BC, in the proleptic Gregorian calendar), a date at which three multi-year cycles started (which are: Indiction, Solar, and Lunar cycles) and which preceded any dates in recorded history. For example, the Julian day number for the day starting at 12:00 UT on January 1, 2000, was 2,451,545.

The **Julian date** [1] (**JD**) of any instant is the Julian day number for the preceding noon in Universal Time plus the fraction of the day since that instant. Julian dates are expressed as a Julian day number with a decimal fraction added. For example, the Julian Date for 00:30:00.0 UT January 1, 2013, is 2,456,293.520833.

The **Julian Period** [1] is a chronological interval of 7980 years; year 1 of the Julian Period was 4713 BC. It has been used by historians since its introduction in 1583 to convert between different calendars. The Julian calendar year 2017 is year 6730 of the current Julian Period. The next Julian Period begins in the year AD 3268.

14.3 POINT OF ARIES [1] (ϒ)

The First Point of Aries is the location of the vernal equinox, and is named for the constellation of Aries. It is one of the two points on the celestial sphere at which the celestial equator meets the ecliptic plane, the other being the First Point of Libra, located exactly 180° from it. Over its year-long journey through the constellations, the Sun crosses the celestial equator from south to north at the First Point of Aries, and from north to south at the First Point of Libra. The First Point of Aries is considered to be the celestial "prime meridian" from which right ascensions are calculated.

The First Point of Aries (also known as the Cusp of Aries) is so called because, when Hipparchus defined it in 130 BCE, it was located in the western extreme of the constellation of Aries, near its border with Pisces and the star γ Aries. Due to the Sun's eastward movement across the sky throughout the year, this western end of Aries was the point at which the Sun entered the constellation, hence the name First Point of Aries.

Due to Earth's axial precession, this point gradually moves westwards at a rate of about one degree every 72 years. This means that, since the time of Hipparchus, it has shifted across the sky by about 30°, and is currently located within Pisces, near its border with Aquarius. Currently, the closest major star to the First Point of Aries is λ Piscium, located at (23h 42m 03s, 01° 46′ 48″).

The Sun now appears in Aries from late April through mid-May, though the constellation is still associated with the beginning of spring.

The Cusp of Aries is important to the fields of astronomy, nautical navigation and astrology. Navigational ephemeris tables record the geographic position of the First Point of Aries as the reference for position of navigational stars. Due to the slow precession of the equinoxes, the Zenith view (above a location) of constellations at a time of year from a given location have slowly walked West (by using solar epochs the drift is known). Tropical zodiac are identically affected and no longer correspond with the constellations (the Cusp of Libra today is located within Virgo), and is the basis for the concept of astrological ages. In sidereal astrology, by contrast, the first point of Aries remains aligned with the Aries constellation.

Coordinated Universal Time [1] **(UTC) and Universal Time** [1] **(UT)**

Universal Time (UT) is a time standard based on Earth's rotation. It is a modern continuation of Greenwich Meridian Time (GMT), i.e., the mean solar time on the Prime Meridian at Greenwich, London, UK. In fact, the expression "Universal Time" is ambiguous (when accuracy of better than a few seconds is required), as there are several versions of it, the most commonly used being Coordinated Universal Time (UTC) and UT1. All of these versions of UT, except for UTC, are based on Earth's rotation relative to distant celestial objects (stars and quasars), but with a scaling factor and other adjustments to make them closer to solar time. UTC is based on International Atomic Time, with leap seconds added to keep it within 0.9 second of UT1.

Examples:

1) Calculate the time en Gliwice (UTC) when in Greenwich it is 12h 13.4 minutes.

Longitude $_{\text{Gliwice}}$ = 18^0 40' 12.28" / 15^0 = 1 hour 14 minutes 40.82 seconds [East –]

- ✓ [HcG] = Civil Hour in Greenwich
- ✓ [HcL] = Civil Hour in Longitude (Gliwice) or Local

[HcG] = [HcL] + L; 12h 13.4' = [HcL] – 1h 14' 40.82"; [HcL] = 13h 28' 4.82"

2) If the sun passes at 12h 13.4 minutes (mid-day) for Greenwich, at what time the sun passes over Gliwice?

It is the same equation but you need consider that Gliwice is on the east of Greenwich and the sun pass before. [HcG] = [HcL] + L.

Now [PMSL] = [HcG] – L, where [PMSL] = [Pass Meridian Upper Local].

[PMSL] = 12h 13.4' – 1h 14' 40.82" = 10h 58' 43,18" (UTC) or Greenwich time.

14.4 EQUATORIAL COORDINATE SYSTEMS [1]

The equatorial coordinate system is a celestial coordinate system widely used to specify the positions of celestial objects. It may be implemented in spherical or rectangular coordinates, both defined by an origin at the centre of the Earth, a fundamental plane consisting of the projection of the Earth's equator onto the celestial sphere (forming the celestial equator), a primary direction towards the vernal equinox, and a right-handed convention.

Equatorial coordinates system in spherical coordinates. The fundamental plane is formed by projection of the Earth's equator onto the celestial sphere, forming the celestial equator (blue). The primary direction is established by projecting the Earth's orbit onto the celestial sphere, forming the ecliptic, and setting up the ascending node of the ecliptic on the celestial equator, the vernal equinox. Right ascensions are measured eastward along the celestial equator from the equinox, and declinations are measured positive northward from the celestial equator – two such coordinate pairs are shown here. Projections of the Earth's north and south geographic poles form the north and south celestial poles, respectively.

The origin at the centre of the Earth means the coordinates are geocentric, that is, as seen from the centre of the Earth as if it were transparent. The fundamental plane and the primary direction mean that the coordinate system, while aligned with the Earth's equator and pole, does not rotate with the Earth, but remains relatively fixed against the background stars. A right-handed convention means that coordinates are positive toward the north and toward the east in the fundamental plane.

Corrections [1]**: Axial precession and Astronomical nutation**

This description of the orientation of the reference frame is somewhat simplified; the orientation is not quite fixed. A slow motion of Earth's axis, precession, causes a slow, continuous turning of the coordinate system westward about the poles of the ecliptic, completing one circuit in about 26,000 years. Superimposed on this is a smaller motion of the ecliptic, and a small oscillation of the Earth's axis, nutation (Fig. 14.7).

Fig. 14.7 Solar aberration. Analemma over Burgos cathedral. Spain

In order to fix the exact primary direction, these motions necessitate the specification of the equinox of a particular date, known as an epoch, when giving a position. The three most commonly used are:

Mean equinox of a standard epoch (usually J2000.0, but may include B1950.0, B1900.0, etc.) is a fixed standard direction, allowing positions established at various dates to be compared directly.

Mean equinox of date is the intersection of the ecliptic of "date" (that is, the ecliptic in its position at "date") with the mean equator (that is, the equator rotated by precession to its position at "date", but free from the small periodic oscillations of nutation). Commonly used in planetary orbit calculation.

True equinox of date is the intersection of the ecliptic of "date" with the true equator (that is, the mean equator plus nutation). This is the actual intersection of the two planes at any particular moment, with all motions accounted for.

A position in the equatorial coordinate system is thus typically specified true equinox and equator of date, mean equinox and equator of J2000.0, or similar. Note that there is no "mean ecliptic", as the ecliptic is not subject to small periodic oscillations.

Corrections must be applied to translate coordinates from equatorial coordinates system to spherical coordinates such as: Right ascension and declination

14.5 SPHERICAL COORDINATES USED IN ASTRONOMY [1]

A star's spherical coordinates are often expressed as a pair, **right ascension** (Fig. 14.8) **and declination**, without a distance coordinate.

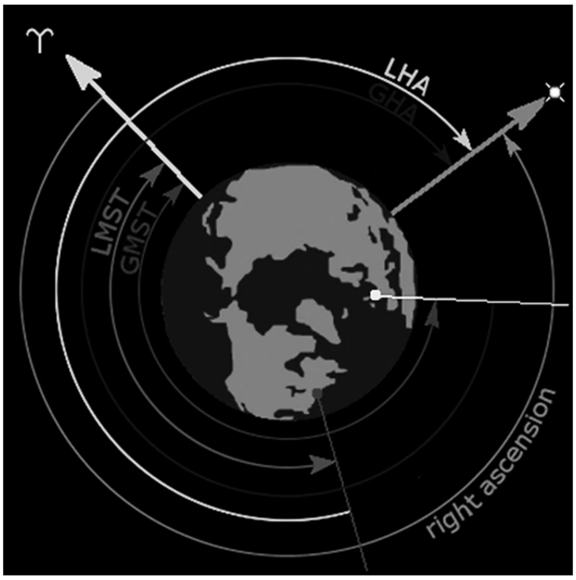

Fig. 14.8 English hour notation angles

The direction of sufficiently distant objects is the same for all observers, and it is convenient to specify this direction with the same coordinates for all. In contrast, in the horizontal coordinate system, a star's position differs from observer to observer based on their positions on the Earth's surface, and is continuously changing with the Earth's rotation.

Telescopes equipped with equatorial mounts and setting circles employ the equatorial coordinate system to find objects. Setting circles in conjunction with a star chart or ephemeris allow the telescope to be easily pointed at known objects on the celestial sphere.

As seen from above the Earth's north pole (Fig. 8), a star's local hour angle (LHA) for an observer near New York (red). Also depicted are the star's right ascension and Greenwich hour angle (GHA), the local mean sidereal time (LMST) and Greenwich mean sidereal time (GMST), [H*G] in Spanish notation. The symbol ♈ identifies the vernal equinox direction.

Declination [1]

The symbol (δ) of declination, (lower case "delta", abbreviated dec.) measures the angular distance of an object perpendicular to the celestial equator, positive to the north, negative to the south. For example, the north celestial pole has a declination of +90°. The origin for declination is the celestial equator, which is the projection of the Earth's equator onto the celestial sphere. Declination is analogous to terrestrial latitude.

Right ascension [1]

The right ascension (AR in Spanish or abbreviated RA in English) (Fig. 8) symbol α, (lower case "alpha) measures the angular distance of an object eastward along the celestial equator from the vernal equinox to the hour circle passing through the object. The vernal equinox point is one of the two where the ecliptic intersects the celestial equator. Analogous to terrestrial longitude, right ascension is usually measured in sidereal hours, minutes and seconds instead of degrees, a result of the method of measuring right ascensions by timing the passage of objects across the meridian as the Earth rotates. There are (360°/24h) = 15° in one hour of right ascension, 24h of right ascension around the entire celestial equator.

When used together, right ascension and declination are usually abbreviated RA/Dec.

For most of Captains, naval officers and me, to follow the concept of clockwise, we are using the "sidereal angle" (A.S. or abbreviated AS) instead of right ascension RA, and the relation is AS $=360^0$ – AR.

Hour angle [1]

Alternatively to right ascension, hour angle (abbreviated HA or LHA. In nautical and Spanish notation [H☼L] *"sun's local hour"* or [H*L] that means *"star's local hour"*), a left-handed system, measures the angular distance of an object westward along the celestial equator from the observer's meridian to the hour circle passing through the object. Unlike right ascension, hour angle is always increasing with the rotation of the Earth. Hour angle may be considered a means of measuring the time since an object crossed the meridian. A star on the observer's celestial meridian is said to have a zero hour angle. One sidereal hour later (approximately 0.9973 solar hours later), the Earth's rotation will carry the star to the west of the meridian, and its hour angle will be +1h. When calculating topocentric phenomena, right ascension may be converted into hour angle as an intermediate step.

Solar equatorial coordinates [1]**. Point of Aries** [1]
To obtain the position of the sun or any star:

- **Step 1.** Where is point of Aries (♈) from Greenwich meridian, also named sidereal time? [HG♈] *"Aries' Greenwich hour"* (Fig. 14.9)
- **Step 2.** What is the value of sidereal angle (AS clockwise criteria [+]) or right ascension (AR [–])?
- **Step 3.** Where is my position (Local longitude)?

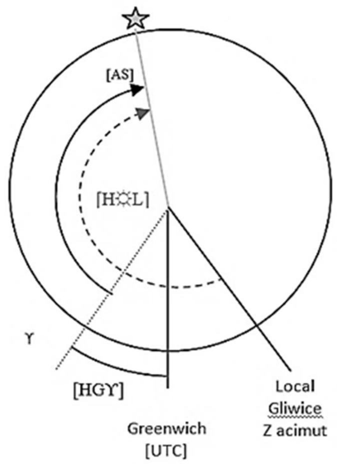

Fig. 14.9 The sun at 00:00

Using the clockwise as (+) positive, the sun's hour in Greenwich is [HG☼] = [HG♈] + AS or
[HG☼] = [HG♈] + (360^0–AR)
The sun's hour in Gliwice is [H☼L] = -L + [HG♈] + AS or [H☼L] = – L + [HG♈] + (360^0–AR) in the Figure 5.
The polar angle **P** is minor of 180^0 always. Remember that **P** is the angle between my local position and the star (sun ☼).
If the sun is at east of me, I have an oriental azimuth (E), and the otherwise is an occidental azimuth (W).
When **P** is west (+), and minus than 180^0, the hour of the sun [H☼L] = **P**.

Solving the triangle with almanac [4]

Well, to solve the triangle we needs: **[Step 1] + [Step 2] + [Step 3]** of above.
And now, we can decide to use mathematics algorithms or nautical [4] almanac (Fig. 14.10).

First of all, I am going to introduce how to use the nautical almanac (Fig. 10) because the values have been tabulated inside, and I will show you how the tables must be used.

Data:

- **Local place:** Gliwice (**N** 50^0 17’ 34.95”; **E** 18^0 40’ 12.28”)
- **Date:** 22/02/2017

To calculate the rise and sunset we need the different between “Hour UTC in Gliwice” and “Hour in Greenwich”.

In the almanac we consider [PMG] or *“Hour of Greenwich Meridian Pass”* or mid-day in Greenwich. [PMG] = 12h 13.4 min and we consider the middle speed of the sun (15^0 in one hour).

The differential time Gliwice and Greenwich should be calculated as given:

$$L = \frac{18^0\ 40'\ 12.28''}{15^0} = 1h\ 14'\ 40.82''\ (East\ -)$$

The sun ☼, pass just over Gliwice [PMSL] at:

[PMSL] = [PMG] – $1h\ 14'\ 40.82''$ = 12h 13.4’ – 1h 14’ 40,82” = 10h 58’ 43,18”

[PMSL] = 10h 58’ 43.18” = 10.97866111 hours

62 miércoles 22 de febrero de 2017

SOL — SD: 16,2′ — PMG: 12h 13,4m

LUNA — SD: 15,0′ — Edad: 25,0d — PHE 4h: 55,1′ 12h: 55,3′ 20h: 55,5′ — PMG: 8h 45m — Ro.: 50m

UT (h)	SOL hG ☉ (° ′)	SOL Dec (° ′)	LUNA hG ☾ (° ′)	Dif	LUNA Dec (° ′)	Dif	Latitud (°)	Principio del crepúsculo Náutico (h m)	Principio del crepúsculo Civil (h m)	Salida de Sol (h m)	Salida de Luna Hora (h m)	Salida de Luna R° (m)	Puesta de Luna Hora (h m)	Puesta de Luna R° (m)
0	176 37,6	−10 12,2	233 04,3	112	−18 50,5	3	60 N	5 47	6 36	7 18	5 03	43	12 28	61
1	191 37,7	11,3	247 34,5	111	50,2	5	58	48	33	13	4 50	44	41	60
2	206 37,7	10,4	262 04,6	111	49,7	5	56	48	31	08	39	44	12 52	59
3	221 37,8	09,5	276 34,7	111	49,2	6	54	47	28	04	29	45	13 02	58
4	236 37,9	08,6	291 04,8	110	48,6	7	52	47	26	7 01	21	45	11	57
5	251 38,0	−10 07,7	305 34,8	110	−18 47,9	8	50	5 47	6 24	6 57	4 13	45	13 19	57
6	266 38,0	−10 06,8	320 04,8	111	−18 47,1	9	45	5 46	6 20	6 50	3 56	46	13 35	56
7	281 38,1	05,9	334 34,9	109	46,2	10	40	45	16	43	42	47	13 49	55
8	296 38,2	04,9	349 04,8	110	45,2	11	35	43	12	38	30	48	14 01	54
9	311 38,3	04,0	3 34,8	109	44,1	12	30	41	09	33	20	48	11	53
10	326 38,4	03,1	18 04,7	110	42,9	13	20	37	6 02	25	3 03	49	28	52
11	341 38,5	−10 02,2	32 34,7	109	−18 41,6	13	10 N	5 31	5 56	6 17	2 47	50	14 43	51
12	356 38,5	−10 01,3	47 04,6	108	−18 40,3	15	0	5 25	5 49	6 10	2 33	50	14 57	50
13	11 38,6	−10 00,4	61 34,4	109	38,8	16	10 S	17	41	6 03	19	51	15 11	50
14	26 38,7	−9 59,5	76 04,3	108	37,2	16	20	5 06	32	5 55	2 04	51	26	49
15	41 38,8	58,5	90 34,1	108	35,6	18	30	4 52	21	46	1 46	52	43	47
16	56 38,9	57,6	105 03,9	108	33,8	18	35	43	14	41	36	52	15 53	47
17	71 38,9	−9 56,7	119 33,7	108	−18 32,0	19	40	4 33	5 06	5 35	1 25	52	16 05	45
18	86 39,0	−9 55,8	134 03,5	108	−18 30,1	21	45	4 19	4 57	5 27	1 11	53	16 18	44
19	101 39,1	54,9	148 33,3	107	28,0	21	50	4 02	45	19	0 54	54	34	43
20	116 39,2	54,0	163 03,0	107	25,9	22	52	3 54	39	15	46	55	42	42
21	131 39,3	53,1	177 32,7	107	23,7	24	54	44	32	11	38	54	16 51	41
22	146 39,4	52,1	192 02,4	107	21,3	24	56	33	25	06	28	55	17 00	41
23	161 39,4	51,2	206 32,1	107	18,9	25	58	20	17	5 00	17	55	11	39
24	176 39,5	−9 50,3	221 01,8		−18 16,4		60 S	3 05	4 08	4 54	0 04	56	17 23	38

Fig. 14.10 Spanish Nautical Almanac

Taking a glance into the almanac we consider the declination of the sun at 10:00 and 11:00 to interpolate my time value.

$\delta_{☼\,10:00} = -10^0\ 3.1'$ and $\delta_{☼\,11:00} = -10^0\ 2.2'$.

In 0.97866111 hours, the sun ☼ decline $0^0\ 0'\ 52.85''$.

$\delta_{☼\,10:58:43.18} = \delta_{☼\,10:00} + 0^0\ 0'\ 52.85'' = -10^0\ 3.1' + 0^0\ 0'\ 52.85'' = -10^0\ 2'\ 13{,}15''$

In my triangle, the side of declination is (Fig. 6):

Sign discussion: Hemisphere, North (+), sun's declination (-)

$\emptyset_{\delta☼} = 90^0 - (-10^0\ 2'\ 13.15'') = 90^0 + 10^0\ 2'\ 13.15'' = 100^0\ 2'\ 13.15''$

The hour in Greenwich of the sun, follow the same way that declination.

$[HG☼]_{10:00} = 326^0\ 38.4'$ and $[HG☼]_{11:00} = 341^0\ 38.5'$. In one hour the speed of the sun have been $15^0\ 0'\ 6''$. In 0.97866111 hours the angle of the sun will be $14^0\ 40'\ 53.57''$ obtained using linear interpolation. And, $[HG☼]_{10:58:43.18} = [HG☼]_{10:00} + 14^0\ 40'\ 53.57'' = 341^0\ 19'\ 17.57''$

Now, we can calculate the azimuth in rise and sunset using the spherical cosine theorem used in Bessel formulas [2].

In the rise and sunset the sun's height a = 0, were $\emptyset_a = 90^0$ the above formula could be written:

$\cos \emptyset_\delta = \cos \emptyset_a \bullet \cos \emptyset_l + \sin \emptyset_a \bullet \sin \emptyset_l \bullet \cos Z$

$\cos \emptyset_\delta = \sin \emptyset_l \bullet \cos Z$

$$Z = \pm \arccos\left[\frac{\cos \emptyset_\delta}{sen \emptyset_l}\right] = = \pm \arccos\left[\frac{\cos(100^0\ 2'\ 13.15'')}{sen(90^0 - 50^0 17' 34.95'')}\right] = \pm 105^0 49' 52.27''$$

- Azimuth $Z_{rise} = 105^0 49' 52.27''$
- Azimuth $Z_{sunset} = 360^0 - 105^0 49' 52.27'' = 254^0\ 10'\ 7{,}73''$

Now, we can calculate the polar angle in rise and sunset using the spherical cosine theorem used in Bessel formulas [2].

$\cos \emptyset_a = \cos \emptyset_\delta \bullet \cos \emptyset_l + \sin \emptyset_\delta \bullet \sin \emptyset_l \bullet \cos P$.

Where:

$$P = \pm \arccos\left[\frac{\cos \emptyset_a - \cos\emptyset_\delta\ \cos\emptyset_l}{sen\emptyset_\delta\ sen\emptyset_l}\right]$$

In our particular case (rise/set) were sun's height a = 0 ➔ $\emptyset_a = 90^0$ the above formula could be written:

$$P = \pm \arccos\left[\frac{-\cos\emptyset_\delta\ \cos\emptyset_l}{sen\emptyset_\delta\ sen\emptyset_l}\right] = \pm \arccos\left[\frac{-1}{\tan \emptyset_\delta\ \tan\emptyset_l}\right]$$

$$P = \pm \arccos\left[\frac{-1}{\tan(100^0\ 2'\ 13.15'')\ \tan(90^0 - 50^0 17' 34.95'')}\right] = 77^0\ 41'\ 37.7''$$

And converting into a time mode we can use the middle speed of the sun $V_☼ = 15^0/h$ or calculated real speed of the sun $15^0 0' 6''$ for today (the different it is no more than 2'' in final results).

In the first case: P_h = 5.179587056 hours (5h 10' 46.51'')

In the second case: P_h = 5.179011591 hours (5h 10' 44.44'')

To obtain the rise and sunset the formula is given by: [PMSL] ± P_h

[PMSL] = 10h 58' 43.18" = 10.97866111 hours (calculated above)

Rise ☼: 10h 58' 43.18" – 5h 10' 44.44" = 5.79965 hours (5h 47' 58,74")

Set ☼: 10h 58' 43.18" + 5h 10' 44.44" = 16.1576726 hours (16h 9' 27,62")

Difference to save energy:

24h – (16h 9' 27.62" – 5h 47' 58.74") = 13h 38' 31.12" (switch on lightered)

About twilights,

In twilights the sun's height is: civil (-6^0), nautical (-12^0) and astronomic (-18^0) and $Ø_{a,\ civil} = 96^0$, $Ø_{a,\ nautical} = 102^0$ y $Ø_{a,\ astronomical} = 108^0$

Solving the triangle (Cosine Theorem in spherical triangles):

$\cos Ø_a = \cos Ø_\delta \bullet \cos Ø_l + \sin Ø_\delta \bullet \sin Ø_l \bullet \cos P$.

Where: $P = \pm \text{arc} \cos \left[\frac{\cos Ø_a - co\ \ Ø_\delta \ cos Ø_l}{sen Ø_\delta \ sen Ø_l}\right]$

$P_{civil} = \pm\ 87.3074723^0 = \pm$ 5h 49' 11.47"

$P_{nautical} = \pm\ 96.73995172^0 =$ 6h 26' 55.01"

$P_{astronomical} = \pm\ 106.1457437^0 =$ 7h 4' 32.15"

Civil twilight: [PMSL] ± $P_{h\ civil}$

Morning: [PMSL] – $P_{h\ civil}$ = 10h 58' 43.18" – 5h 49' 11.47" = 5h 9' 31,71"

Noon: [PMSL] + $P_{h\ civil}$ = 10h 58' 43.18" + 5h 49' 11.47" = 16h 47' 54,65"

Difference:

24h – (16h 47' 5.65" – 5h 9' 31.71") = 12h 21' 37.06" (switch on lightered)

Nautical twilight: [PMSL] ± $P_{h\ nautical}$

Morning: [PMSL] – $P_{h\ nautical}$ = 10h 58' 43,18" – 6h 26' 55.01" = 4h 31' 48,17"

Noon: [PMSL] + $P_{h\ nautical}$ = 10h 58' 43,18" + 6h 26' 55.01" = 17h 25' 38,19"

Difference:

24h – (17h 25' 38.19" – 4h 31' 48.17") = 11h 6' 9.98" (switch on lightered)

Astronomical twilight: [PMSL] ± $P_{h\ astronomical}$

Morning: [PMSL] – $P_{h\ astronomical}$ = 10h 58' 43,18" – 7h 4' 32.15" = 3h 54' 11,03"

Noon: [PMSL] + $P_{h\ astronomical}$ = 10h 58' 43,18" + 7h 4' 32.15" = 18h 54' 11,03"

14.6 SAVING ELECTRIC POWER

We should save energy working over rise and sunset (calculated above).

24h – (16h 9' 27.62" – 5h 47' 58.74") =
13h 38' 31.12" (switch on lightered)

a) *Civil:*

24h – (16h 47' 5.65" – 5h 9' 31.71") = 12h 21' 37.06" (switch on lightered)

Saved energy: 13h 38' 31.12" – 12h 21' 37.06" = 1h 16' 54.06" per light.

b) *Nautical:*

24h – (17h 25' 38.19" – 4h 31' 48.17") = 11h 6' 9.98" (switch on lightered)

Saved energy: 13h 38' 31.12" – 11h 6' 9.98" = 2h 32' 21.14" per light

It can be very interesting to adjust the height of the Sun which lights should be lit.

Solutions with mathematics algorithms [1] **using spreadsheet** (Fig. 14.11 and Fig. 14.12)

DÍA	22	HORA	0	Zona GMT	Dif. Horaria [HZ]
MES	2	MINUTO	0	1	1
AÑO	2017	SEGUNDO	0	1 h 14,68'	1,244671852
	(°) grad	(') min	('') seg	(°) grad	Signo Hemisf.
Latitud	50	17	34,95	50,29304167	1
Longitud	18	40	12,28	18,67007778	

Hora	Fracción día	Día Juliano	Fracc. Siglo Juliano	T (periodo)	[HGY] Tiempo sidereo Θ_0 h
0:00:00	22,00000000	2457806,50000000	0,1714305270	0,171430527	6272,0916138
12:13:30	22,50937500	2457807,00937500	0,1714444730	0,171444473	6272,5936779
10:58:49	22,45751157	2457806,95751157	0,1714430530	0,171443053	6272,5425589

[P⓪MSL] (h)	δ⓪(°)$_{PMSL}$	[H⓪]$_{PMSL}$	Azimut ORTO
10,98030514	-10,03614362	341,3442396	105,8298359
10h 58,8'	-10° 2,17'	341° 20,7'	105° 49,8'
PMG (h)	δ⓪(°)$_{PMG}$	[HG⓪]$_{PMG}$	Azimut OCASO
12,224977	-10,01717984	0,01677666	254,1701641
12h 13,5'	-10° 1,03'	0° 1'	254° 10,2'

Fig. 14.11 MS Excel. Sidereal time at 00:00, [PMG] and [PMSL]. Declination and Azimuths

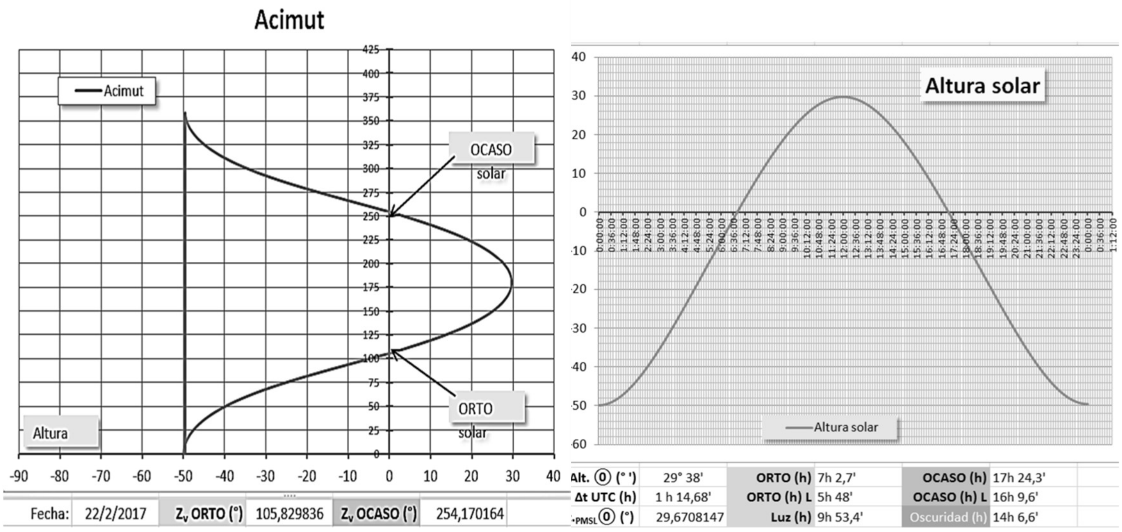

Fig. 14.12 MS Excel. Sun height and azimuth

The realization of numbers calculations can be tedious if they are made by hand. For this reason, a worksheet can be used to introduce the mathematical formulation that allows solving our problem.

It would be advisable to read the book[1] Astronomical Algorithms described in bibliography. For example:

To calculate the Julian day (JD_{2000}) at 00:00 h February 22th, 2017 we can use this MS Excel algorithm made by myself: JD = DATE–DATE(2000;1;1)+2451544,5

But I think that the problem may be higher depending on the complexity of the calculation.

For example, to calculate the sidereal time (Aries) Θ_0[1] = [HGϒ] at 00:00 h = $[HG\Upsilon]_{00:00}$

= 100.46061837+36000.770053608*T+0.000387933*T^2) –T^3/38710000

Where:

The period T [1] = (Julian Day –2451545)/36525

We could see graphics Sun height and azimuth

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Date of submission of the article to the Editor: 02.2017
Date of acceptance of the article by the Editor: 04.2017

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SAVING ENERGY IN LIGHT

Abstract: *One of the main problems to consider by engineering nowadays is "to manage the energy effectively", and light up the city or external areas in a mine. It is being very expensive. The engineer could be manage the switch on/off over streetlight giving a timetable per day, month and year and calculate the "saved electric power". You can establish future predictions about savings in function of mathematics parameters based on the solar ecliptic path. You can decrease the light pollution effects. You can manage and use, different switch on/off timetables per groups of light lines.*

Key words: ***s****olar calculations, saving energy, rise and sunset calculations, twilight calculations*

OSZCZĘDNOŚĆ ENERGII W OŚWIETLENIU

Streszczenie: *Jedną z głównych kwestii poruszanych w inżynierii w dzisiejszych czasach jest „efektywne zarządzanie energią", i oświetlenie miasta lub zewnętrznych obszarów kopalni. Jest to bardzo kosztowne. Inżynier może zarządzać przełącznikiem włącz/wyłącz dla latarni miejskich, ustalając harmonogram na dzień, miesiąc i rok oraz obliczyć „zaoszczędzoną energię elektryczną". Można ustalić przyszłe prognozy dotyczące oszczędności w zależności od parametrów matematycznych opartych na ekliptycznej ścieżce słońca. Można zmniejszyć skutki zanieczyszczenia świetlnego, a także zarządzać i wykorzystywać inne harmonogramy przełącznika włącz/wyłącz na grupach linii świetlnych.*

Słowa kluczowe: *obliczenia słoneczne, oszczędność energii, obliczenia wschodów i zachodów słońca, obliczenia zmierzchu*